Comment on "Vacuum electron acceleration by coherent dipole radiation"

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Troha *et al.* [Phys. Rev. E **60**, 926 (1999)] put forward a generalized Lawson-Woodward theorem in the study of laser accelerations. We point out that one of the assumptions used in their proof does not stand on a solid physical ground and that it is possible for electrons to obtain net energy gains from a plane-wave laser pulse in vacuum even if the radiation reaction effects are neglected.

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Whether an electron can be accelerated by lasers in vacuum has been discussed for decades [1–6]. According to the Lawson-Woodward (LW) theorem, an electron can never obtain a net energy gain from a plane wave field under certain conditions, which have been summarized by Esarey in [7] to be:

(i) the laser field is in vacuum with no walls or boundaries present,

(ii) the electron is highly relativistic ($v \simeq c$) along the acceleration path,

(iii) no static or magnetic fields are present,

(iv) the region of interaction is infinite,

(v) the force $-e\vec{v}\times\vec{B}$ is neglected.

Therefore, to achieve a nonzero energy gain by using laser fields in vacuum, one or more of the above assumptions must be violated. Similar conclusions can also be found in Ref. [5].

Recently, Troha *et al.* [8] put forward a generalized version of LW, which claims that the electron can get no energy gain so long as plane waves are considered and the radiation reaction effects are neglected. But one of the assumptions leading to their conclusion does not stand on a solid physical ground. For convenience of the discussion, we first review the electron dynamics in a plane wave [8], which propagates along the z axis with the four-potential,

$$A_\mu(\phi)=[\varphi,\vec{A}_\perp(\phi),A_z],\quad \varphi=A_z=0,\quad \phi=k_\mu x^\mu(\tau), \tag{1}$$

in which τ is the proper time and $k^\mu=(\omega,\vec{k})$ the four-wave number. By solving the Newton-Lorentz equation for the motion of the electron,

$$\frac{d\vec{P}}{dt}=-e(\vec{E}+\vec{v}\times\vec{B}), \tag{2}$$

we obtain [8]

$$P_z=\gamma_0\left[\beta_0+\frac{\vec{A}_\perp^2(\phi)}{2}(1+\beta_0)\right], \tag{3}$$

$$\gamma=\gamma_0\left[1+\frac{\vec{A}_\perp^2(\phi)}{2}(1+\beta_0)\right]. \tag{4}$$

Then as concluded in Ref. [8], by using $\lim_{\phi\to\pm\infty}\vec{A}_\perp(\phi)=0$, the electron net energy gain is zero from Eq. (4).

But the condition $\lim_{\phi\to\pm\infty}\vec{A}_\perp(\phi)=0$ is not a necessary physical requirement. A more reasonable condition should be $\lim_{\phi\to\pm\infty}\vec{E}_\perp(\phi)=0$, which however does not lead naturally to $\lim_{\phi\to\pm\infty}\vec{A}_\perp(\phi)=0$. To see this, we express $\vec{A}_\perp(\phi)$ by $\vec{E}_\perp(\phi)$ as follows:

$$\vec{A}_\perp(\phi)=\vec{A}_\perp(-\infty)-\frac{1}{\omega}\int_{-\infty}^{\phi}\vec{E}_\perp(\phi)d\phi. \tag{5}$$

As an example, we assume $\vec{E}_\perp(\phi)=E_0\exp[-\phi^2/2w_\phi^2]\cos\phi$ with w_ϕ to be the pulse length in units of $1/k$. Then

$$\vec{A}_\perp(+\infty)-\vec{A}_\perp(-\infty)=-\frac{E_0}{\omega}\sqrt{2\pi}w_\phi e^{-w_\phi{}^2/2}\vec{e}_\perp, \tag{6}$$

which is nonzero except that $w_\phi\to\infty$ or $w_\phi=0$. The maximum value of $|\vec{A}_\perp(+\infty)-\vec{A}_\perp(-\infty)|$ occurs when $w_\phi=1$. This is just the so-called subcycle laser accelerations, which has been put forward by Rau *et al.* [9] and studied by Cheng *et al.* [10]. From here, we can see that it is possible for the electrons to gain energy from the plane-wave pulses even if the radiation reaction effects are neglected.

To give a more general consideration of the electron dynamics in the plane-waves pulse, we start directly from the electro-magnetic fields, $E_x=B_y=E_0 f(\phi)\cos(\phi)$, instead of the corresponding four-potential. By solving Eq. (2), we can find the energy gain of the electron in units of $m_e c^2$ (m_e is the rest mass of the electron) to be

$$\Delta\gamma=\gamma-\gamma_0=\frac{A^2-2A\tilde{P}_{x0}}{2(\gamma_0-\tilde{P}_{z0})}, \tag{7}$$

in which $\tilde{P}_{x0}$ and $\tilde{P}_{z0}$ are the electron initial transverse and longitudinal momentum in units of $m_e c^2$ and

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$$A=Q_0\int_{-\infty}^{+\infty}f(\phi)\cos(\phi+\phi_0)d\phi=Q_0\,\mathrm{Re}(\tilde{f}(\eta)|_{\eta=1}),\tag{8}$$

where $Q_0=eE_0/m_e\omega c$, $\tilde{f}(\eta)$ the Fourier component of $f(\phi)$ defined as follows:

$$\tilde{f}(\eta)=\int_{-\infty}^{+\infty}f(\phi)e^{i\eta\phi}d\phi.\tag{9}$$

By inverse Fourier transformations, $f(\phi)$ can be expressed as

$$f(\phi)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\tilde{f}(\eta)e^{-i\phi\eta}d\eta.\tag{10}$$

Because $f(\phi)$ is a real function, the real and imaginary parts of $\tilde{f}(\eta)$, i.e., $\mathrm{Re}(\tilde{f}(\eta))$ and $\mathrm{Im}(\tilde{f}(\eta))$, must satisfy

$$\int_{-\infty}^{+\infty}(\mathrm{Re}(\tilde{f}(\eta))\sin(\phi\eta)-\mathrm{Im}(\tilde{f}(\eta))\cos(\phi\eta)d\eta=0.\tag{11}$$

To meet the above equation, a natural option is to choose $\mathrm{Im}(\tilde{f}(\eta))$ to be zero and $\mathrm{Re}(\tilde{f}(\eta))$ to be symmetric with $\eta=0$. Then $f(\phi)$ is also symmetric with respect to $\phi=0$ according to Eq. (10).

Since $\Delta\gamma\sim A^2$, for the estimation of the magnitude of A, we assume that $f(\phi)$ varies appreciably over the scale length w_ϕ, i.e., we replace $f(\phi)$ by $F(\phi/w_\phi)$ in the following. Then Eq. (8) can be rewritten as $(f(\phi)=F(\phi/w_\phi))$,

$$A=Q_0\int_{-\infty}^{+\infty}F\left(\frac{\phi}{w_\phi}\right)e^{i\phi}d\phi=w_\phi Q_0\int_{-\infty}^{+\infty}F(\chi)e^{iw_\phi\chi}d\chi.\tag{12}$$

Now we consider two limiting cases. One is with $w_\phi\ll 1$, from which it can be easily found that $A\sim w_\phi$. The other is with $w_\phi\gg 1$. Then if $F(\chi)$ has no singularities on the real axis, A is exponentially small. To see this, assuming χ_1 to be the distance from the real axis to the nearest singularity of $F(\chi)$ in the upper half plane, we may estimate

$$A\sim w_\phi e^{-\chi_1 w_\phi}\quad(\phi_1 w_\phi\gg 1).\tag{13}$$

If the nth derivative of $F(\chi)$, i.e., $F^{(n)}(\chi)$, has discontinuities at $\chi=\pm a$ on the real axis while all other lower derivatives are continuous, we can find that A decreases according to a power law, i.e., $A\sim 1/w_\phi^n$. To prove this, we write

$$A=w_\phi Q_0\left[\int_{-\infty}^{a}F(\chi)e^{iw_\phi\chi}d\chi+\int_{-a}^{+a}F(\chi)e^{iw_\phi\chi}d\chi+\int_{a}^{+\infty}F(\chi)e^{iw_\phi\chi}d\chi\right],\tag{14}$$

and integrate $n+1$ times by parts. Then

$$A\approx\begin{cases}2[-i^nQ_0\sin(w_\phi a)\Delta(F^{(n)}(a))]/w_\phi^n & (a\neq 0,\ w_\phi\gg 1),\\ 2[i^{n+1}Q_0\Delta(F^{(n)}(0))]/w_\phi^n & (a=0,\ w_\phi\gg 1),\end{cases}\tag{15}$$

where the symmetric property of $f(\phi)$ has been used and

$$\Delta(F^{(n)}(a))=\lim_{\varepsilon\to 0}(F^{(n)}(a+\varepsilon)-F^{(n)}(a-\varepsilon)).\tag{16}$$

From the above discussions, we can see when $w_\phi\ll 1$ or $w_\phi\gg 1$, A will decrease to zero according to different scaling laws. Usually, such transition from $w_\phi\ll 1$ to $w_\phi\gg 1$ happens when w_ϕ is of unit magnitude [11]. Thus, in order to accelerate the electrons with plane-wave pulses, it is very necessary to use subcycle pulses, or at least some pulses with subcycle properties as discussed in [10].

In summary, in this Comment, we have pointed out that the generalized Lawson-Woodward theorem mentioned by Troha *et al.* [8] does not stand if the basis of the proof is examined more carefully. Therefore, it is possible for electrons to obtain net energy gain from plane-wave laser pulses. Furthermore, the general electron dynamics is also discussed briefly to show that only subcycle pulses or pulses with subcycle properties can be used to give the electron a large energy gains.

Finally, it should be mentioned that such plane-wave pulse is only an ideal model of the realistic fields. To study what will happen when the diffraction effect is included, we have also built a realistic subcycle pulse model to make the simulations. The results are published in a recent paper. The conclusion is that when the beam width is very wide, the electron energy gain is very similar to the results gotten by the plane-wave model.

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